Physical Security of Code-based Cryptosystems based on the Syndrome Decoding Problem

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Context

2016: NIST called for proposals for **post-quantum cryptography** algorithms

- Key encapsulation mechanisms . . . . 🔒
- Digital signature . . . . 📝

Three rounds:

2017	Round 1: 69 candidates,
2019	Round 2: 26 candidates,
2020	**Round 3:** 7 finalists (+8 alternate).

Key-Encapsulation Mechanisms finalists:

- Lattice-based: Kyber, NTRU and Saber,
- **Code-based**: Classic McEliece [1]

Research challenges

- *“More hardware implementations”*
- *“Side-channel attacks / resistant implem.”*

Dustin Moody (NIST), PKC 2022

[1] M. R. Albrecht, D. J. Bernstein, T. Chou, et al. *Classic McEliece*. Tech. rep. National Institute of Standards and Technology, 2020

Classic McEliece

Classic McEliece : Niederreiter cryptosystem

Classic McEliece is based on the **Niederreiter cryptosystem** [2]:

- `KeyGen`(n, k, t) = (pk, sk)

  $\mathbf{H}$: parity-check matrix of $\mathcal{C}$, an $[n, k]$ linear code with an efficient decoding algorithm that can correct up to t errors
  $\mathbf{S}$: random invertible matrix of size $n - k$
  $\mathbf{P}$: random permutation matrix of size n
  Compute $\mathbf{H}_{pub} = \mathbf{SHP}$
  pk = $(\mathbf{H}_{pub}, t)$ `/* public key */`
  sk = $(\mathbf{S}, \mathbf{H}, \mathbf{P})$ `/* secret key */`

- `Encrypt`$(\mathbf{m}$, pk$)$ = $\mathbf{s}$

  Encode $\mathbf{m}$ into a constant-weight vector $\mathbf{e}$ of Hamming weight t
  Compute the syndrome $\mathbf{s} = \mathbf{H}_{pub}\mathbf{e}$

[2] H. Niederreiter. “Knapsack-Type Cryptosystems and Algebraic Coding Theory”. In: *Problems of Control and Information Theory* 15.2 (1986), pp. 159–166.

Security

The security of the Niederreiter cryptosystem relies on the **syndrome decoding problem**.

Syndrome decoding problem

Input: a binary matrix $\mathbf{H} \in \mathbb{F}_2^{(n-k)\times n}$

a binary vector $\mathbf{s} \in \mathbb{F}_2^{n-k}$

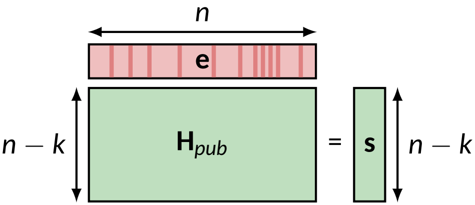
a scalar $t \in \mathbb{N}^+$

Output: a binary vector $\mathbf{x} \in \mathbb{F}_2^n$ with a Hamming weight $\mathrm{HW}(\mathbf{x}) \leq t$ such that : $\mathbf{Hx} = \mathbf{s}$

Known to be an **NP-hard** problem [3].

[3] E. R. Berlekamp, R. J. McEliece, and H. C. A. van Tilborg. "On the inherent intractability of certain coding problems (Corresp.)". In: *IEEE Transactions on Information Theory* 24.3 (1978), pp. 384–386.

Classic McEliece parameters



n	k	t	Equivalent bit-level security
3488	2720	64	128
4608	3360	96	196
6688	5024	128	256
6960	5413	119	256
8192	6528	128	256

The public key $\mathbf{H}_{pub}$ is **very large**!

Hardware implementations

Implementations on embedded systems are now feasible : [4] [5] [6]
Reference hardware target : ARM® Cortex®-M4

Several **strategies** to store the (very large) keys :

- Streaming,
- Use a structured code,
- Use a very large microcontroller.

New threats

That makes them vulnerable to **physical attacks** (fault injection & side-channel analysis)

[4] S. Heyse. "Low-Reiter: Niederreiter Encryption Scheme for Embedded Microcontrollers". In: *International Workshop on Post-Quantum Cryptography*. Vol. 6061. Darmstadt, Germany: Springer, May 2010, pp. 165–181.

[5] J. Roth, E. G. Karatsiolis, and J. Krämer. "Classic McEliece Implementation with Low Memory Footprint". In: *CARDIS*. vol. 12609. Virtual Event: Springer, Nov. 2020, pp. 34–49.

[6] M.-S. Chen and T. Chou. "Classic McEliece on the ARM Cortex-M4". In: *IACR Transactions on Cryptographic Hardware and Embedded Systems* 2021.3 (2021), pp. 125–148.

“Modified” syndrome decoding problem

Syndrome decoding problem

Binary syndrome decoding problem (Binary SDP)

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$\mathbb{N}$ syndrome decoding problem ($\mathbb{N}$-SDP)

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$\mathbb{N}$ syndrome decoding problem ($\mathbb{N}$-SDP)

Input: a binary matrix $\mathbf{H} \in \{0,1\}^{(n-k)\times n}$
a ~~binary~~ vector $\mathbf{s} \in \mathbb{N}^{n-k}$ ← How do we get this integer syndrome?
a scalar $t \in \mathbb{N}^+$

Output: a binary vector $\mathbf{x} \in \{0,1\}^n$ with a Hamming weight $\mathsf{HW}(\mathbf{x}) \leq t$ such that : $\mathbf{Hx} = \mathbf{s}$

Physical attack #1: Fault injection

Syndrome computation : $\mathbf{Hx} = \mathbf{s}$

We target the **syndrome computation**: $\mathbf{s} = \mathbf{H}_{pub}\mathbf{e}$

Matrix-vector multiplication performed over $\mathbb{F}_2$

Algorithm 1 Schoolbook matrix-vector multiplication over $\mathbb{F}_2$

```
function MAT_VEC_MULT_SCHOOLBOOK(matrix, vector)
  for row ← 0 to $n - k - 1$ do
    syndrome[row] = 0                                              ▷ Initialisation
  for row ← 0 to $n - k - 1$ do
    for col ← 0 to $n - 1$ do
      syndrome[row] ^= matrix[row][col] & vector[col]      ▷ Multiplication and addition
  return syndrome
```

Laser fault injection attack on the schoolbook matrix-vector multiplication

Targeting the XOR operation, considering the Thumb instruction set.

bits	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
EORS: Rd = Rm ⊕ Rn	0	1	0	0	0	0	0	0	0	1	Rm			Rdn		
EORS: R1 = R0 ⊕ R1	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	1

Laser fault injection in Flash memory : **mono-bit, bit-set fault model** [7].

[7] A. Menu, J.-M. Dutertre, J.-B. Rigaud, et al. “Single-bit Laser Fault Model in NOR Flash Memories: Analysis and Exploitation”. In: *FDTC*. Milan, Italy: IEEE, Sept. 2020, pp. 41–48.

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Laser fault injection in Flash memory : **mono-bit, bit-set fault model** [7].

ADCS: R1 = R0 + R1	0	1	0	0	0	0	0	1	0	1	0	0	0	0	0	1

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Laser fault injection in Flash memory : **mono-bit, bit-set fault model** [7].

ADCS: R1 = R0 + R1	0	1	0	0	0	0	0	1	0	1	0	0	0	0	0	1

Outcome: switching from $\mathbb{F}_2$ to $\mathbb{N}$

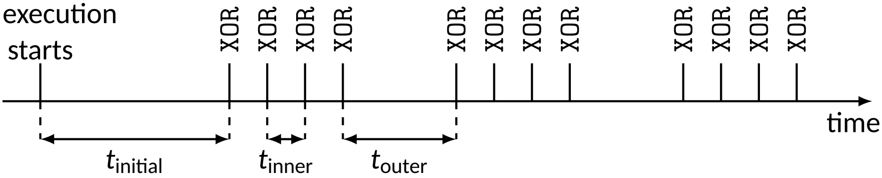
The exclusive-OR (addition over $\mathbb{F}_2$) is turned into an **addition with carry** (addition over $\mathbb{N}$)

[7] A. Menu, J.-M. Dutertre, J.-B. Rigaud, et al. “Single-bit Laser Fault Model in NOR Flash Memories: Analysis and Exploitation”. In: *FDTC*. Milan, Italy: IEEE, Sept. 2020, pp. 41–48.

Multiple faults

Three independent delays must be tuned to fault the full matrix-vector multiplication:

- t_{initial} : **initial** delay before the multiplication starts
- t_{inner} : delay in the **inner** `for` loop
- t_{outer} : delay in the **outer** `for` loop



Outcome

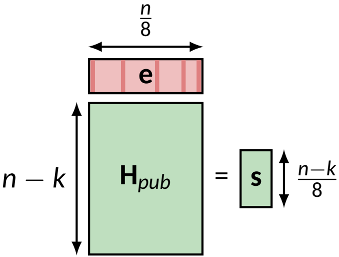
After $n.(n-k)$ faults, we get a **faulty syndrome s** $\in \mathbb{N}^{n-k}$

Packed matrix-vector multiplication

Objection: the schoolbook matrix-vector multiplication algorithm is **highly inefficient**!
Each **machine word** stores only **one bit**: a **lot** of memory is wasted.

Algorithm 2 Packed matrix-vector multiplication

```
function Mat_vec_mult_packed(mat, vector)
  for row ← 0 to $((n - k)/8 - 1)$ do
    syn[row] = 0                                  ▷ Initialisation
  for row ← 0 to $(n - k - 1)$ do
    $b = 0$
    for col ← 0 to $(n/8 - 1)$ do
      b ^= mat[row][col] & vector[col]
    $b$ ^= $b >> 4$
    $b$ ^= $b >> 2$                               ▷ Exclusive-OR folding
    $b$ ^= $b >> 1$
    $b$ &= 1                                      ▷ LSB extraction
    syn[row/8] |= $b$ << ($row\%8$)               ▷ Packing
  return syn
```



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    $b$ ^= $b$ >> 1
    $b$ &= 1                                  ▷ LSB extraction
    syn[row/8] |= $b$ << ($row$%8)            ▷ Packing
  return syn
```

Attack not directly applicable here

We suggested the following strategy (**admittedly not feasible**):

- Prematurely exit the **inner** `for` loop to keep only one byte
- Reverse the exclusive-OR folding permutation over $\mathbb{F}_2^8$
- Mask with `0xFF` instead of 1
- For bit packing:
  - Turn shift into `CMP`
  - Prematurely exit the **outer** `for` loop to keep only one byte

Physical attack #2: Side-channel analysis

Side-channel analysis to obtain the integer syndrome

Algorithm 2 Packed matrix-vector multiplication

```
...
for col ← 0 to $(n/8 - 1)$ do
    b ^= mat[row][col] & vector[col]
...
```

```
b = 00000000

b = 00000000

b = 00001000

b = 00001000

b = 00001010
```

Side-channel analysis to obtain the integer syndrome

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```
...
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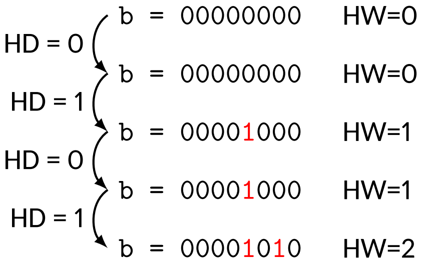
...
```



b = 00000000 HW=0

HD = 0

b = 00000000 HW=0

HD = 1

b = 00001000 HW=1

HD = 0

b = 00001000 HW=1

HD = 1

b = 00001010 HW=2

Side-channel analysis to obtain the integer syndrome

Algorithm 2 Packed matrix-vector multiplication

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...
for col ← 0 to $(n/8 - 1)$ do
    b ^= mat[row][col] & vector[col]
...
```
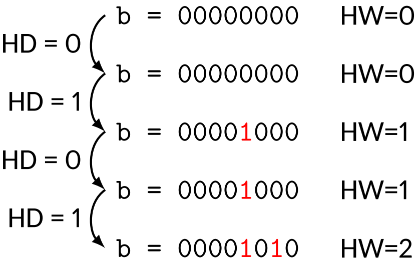



HD = 0 (b = 00000000 HW=0 → b = 00000000 HW=0)

HD = 1 (b = 00000000 HW=0 → b = 00001000 HW=1)

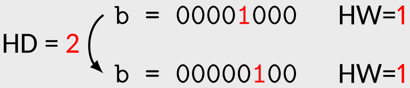
HD = 0 (b = 00001000 HW=1 → b = 00001000 HW=1)

HD = 1 (b = 00001000 HW=1 → b = 00001010 HW=2)

Integer syndrome from Hamming distances or Hamming weights

$$s_j = \sum_{i=1}^{\frac{n}{8}-1} \mathrm{HD}(\mathbf{b}_{j,i}, \mathbf{b}_{j,i-1})$$

$$= \sum_{i=1}^{\frac{n}{8}-1} \left| \mathrm{HW}(\mathbf{b}_{j,i}) - \mathrm{HW}(\mathbf{b}_{j,i-1}) \right| \quad \text{if } \mathrm{HD}(\mathbf{b}_{j,i}, \mathbf{b}_{j,i-1}) \leq 1$$

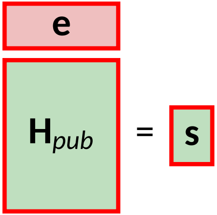


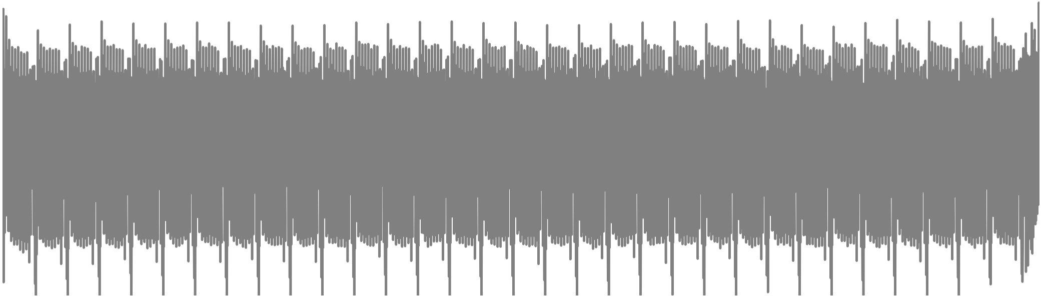
HD = 2 (b = 00001000 HW=1 → b = 00000100 HW=1)

Happens if:
$\mathrm{HW}(\texttt{mat[r][c] \& e\_vec[c]}) > 1$
Unlikely, since $\mathrm{HW}(\mathbf{e}) = t$ is low.

Side-channel analysis for Hamming weight recovery

$$\mathbf{s} = \mathbf{H}_{pub}\mathbf{e}$$





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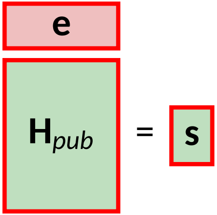
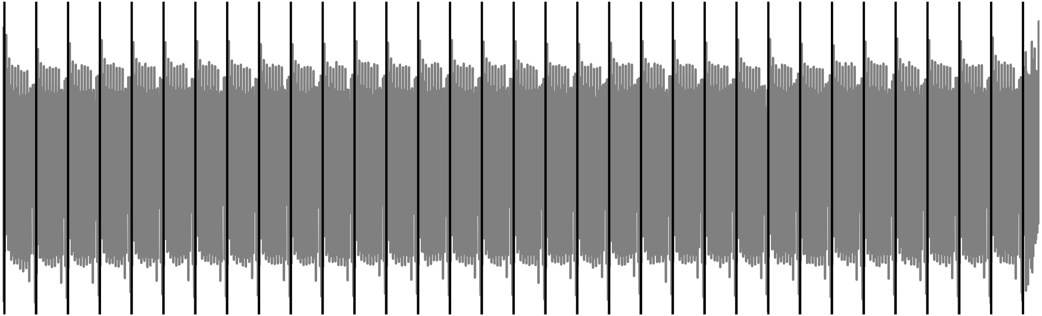
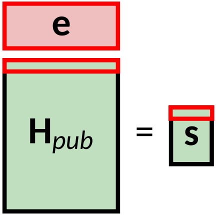
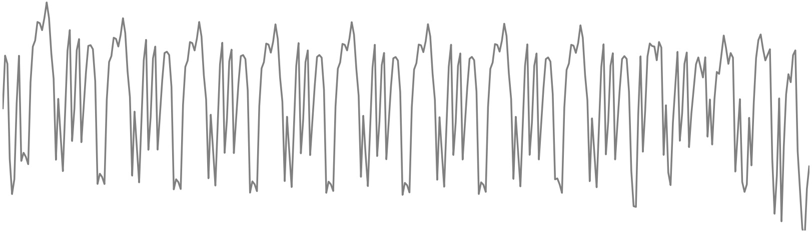




Side-channel analysis for Hamming weight recovery

$$\mathbf{s}_j = \mathbf{H}_{pub_{[j,]}}\mathbf{e}$$





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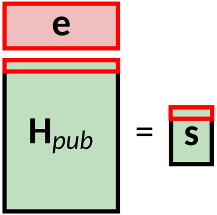
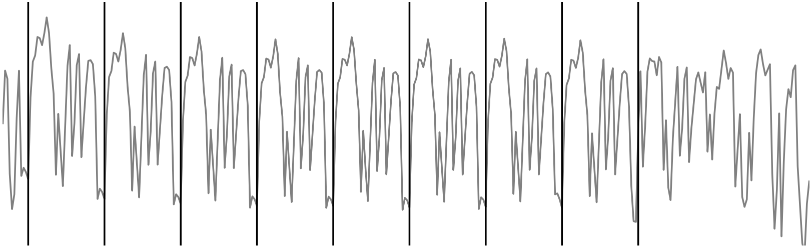


$\mathbf{e}$

$\mathbf{H}_{pub}$ = $\mathbf{s}$



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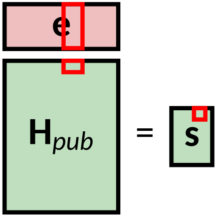
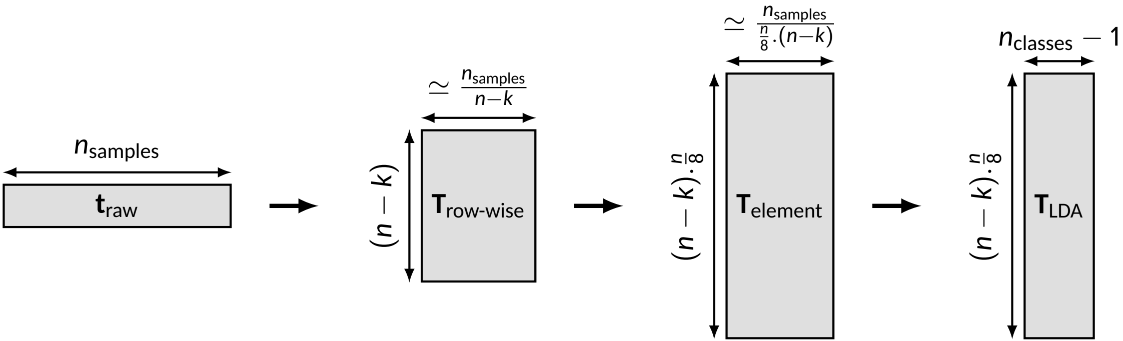
$$b\ \hat{}= \mathbf{H}_{pub_{[j,i]}}\mathbf{e}_i$$



Trace(s) reshaping process


n_{samples}
$\mathbf{t}_{\text{raw}}$
$\simeq \frac{n_{\text{samples}}}{n-k}$
$(n-k)$
$\mathbf{T}_{\text{row-wise}}$
$\simeq \frac{n_{\text{samples}}}{\frac{n}{8}.(n-k)}$
$(n-k).\frac{n}{8}$
$\mathbf{T}_{\text{element}}$
$n_{\text{classes}} - 1$
$(n-k).\frac{n}{8}$
$\mathbf{T}_{\text{LDA}}$

Training phase

- Linear Discriminant Analysis (LDA) for dimensionality reduction,
- From a **single** trace, we get $(n-k) \times \frac{n}{8}$ training samples $n = 8192$ → more than 1.7×10^6
- Fed to a **single** Random Forest classifier (`sklearn.ensemble.RandomForestClassifier`)

Random Forest classifier

Random Forest classifier training:

- Hamming weight:
  - $> 99.5\,\%$ test accuracy,
- Hamming distance:
  - $\approx 80\,\%$ test accuracy.

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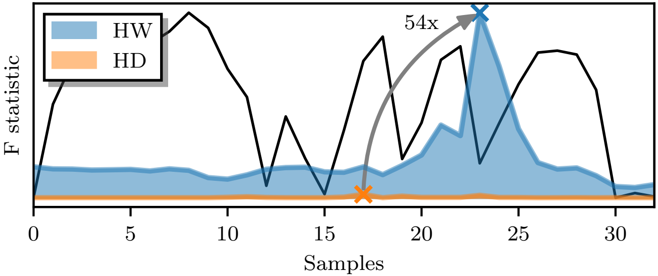

Outcome

- We can recover the **Hamming weight** very accurately,
- but **not the Hamming distance**...
- We can compute a *slightly innacurate* integer syndrome.

Exploiting the integer syndrome

Exploiting the integer syndrome

Option 1: Consider $\mathbf{H}_{pub}\mathbf{e} = \mathbf{s}$ as an **optimization problem** and solve it.

$\mathbb{N}$ syndrome decoding problem ($\mathbb{N}$-SDP)

Input: a matrix $\mathbf{H}_{pub} \in \mathcal{M}_{n-k,n}(\mathbb{N})$ with $h_{i,j} \in \{0,1\}$ for all i, j
a vector $\mathbf{s} \in \mathbb{N}^{n-k}$
a scalar $t \in \mathbb{N}^{+}$

Output: a vector $\mathbf{e}$ in $\mathbb{N}^n$ with $x_i \in \{0,1\}$ for all i
and with a Hamming weight HW($\mathbf{x}$) $\leq t$ such that : $\mathbf{H}_{pub}\mathbf{e}$ = $\mathbf{s}$

ILP problem

Let $\mathbf{b} \in \mathbb{N}^n$, $\mathbf{c} \in \mathbb{N}^m$ and $\mathbf{A} \in \mathcal{M}_{m,n}(\mathbb{N})$
We have the following optimization problem:

$$\min\{\mathbf{b}^T\mathbf{x} \mid \mathbf{A}\mathbf{x} = \mathbf{c}, \mathbf{x} \in \mathbb{N}^n, \mathbf{x} \geq 0\}$$

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Can be solved by **integer linear programming.**
With `Scipy.optimize.linprog`:

- $n = 256$: 0.2 s
- $n = 8192$:

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With `Scipy.optimize.linprog`:

- $n = 256$: 0.2 s
- $n = 8192$: $\approx$ 5 min...

Does not handle errors in **s** well...

Exploiting the integer syndrome

Option 2 (*Quantitative Group Testing* [8]): which columns of $\mathbf{H}_{pub}$ "contributed" to the syndrome.

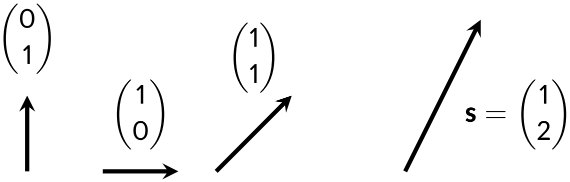
[8] U. Feige and A. Lellouche. "Quantitative Group Testing and the rank of random matrices". In: *CoRR* abs/2006.09074 (2020). arXiv: `2006.09074`.

Exploiting the integer syndrome

Option 2 (*Quantitative Group Testing* [8]): which columns of $\mathbf{H}_{pub}$ “contributed” to the syndrome.

Example: $t = 2 = \mathrm{HW}(\mathbf{e})$

$$\mathbf{H}_{pub}\mathbf{e} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} . \mathbf{e} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$


$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
$\mathbf{s} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

[8] U. Feige and A. Lellouche. “Quantitative Group Testing and the rank of random matrices”. In: *CoRR* abs/2006.09074 (2020). arXiv: 2006.09074.

Exploiting the integer syndrome

Option 2 (*Quantitative Group Testing* [8]): which columns of $\mathbf{H}_{pub}$ "contributed" to the syndrome.

Example: $t = 2 = \mathrm{HW}(\mathbf{e})$

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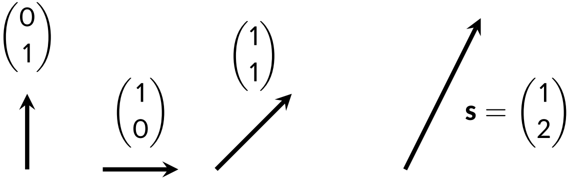


$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
$\mathbf{s} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Score function

The dot product can be used to compute a "score" for every column:

$$\psi(i) = \mathbf{H}_{pub[,i]} \cdot \mathbf{s} + \bar{\mathbf{H}}_{pub[,i]} \cdot \bar{\mathbf{s}} \qquad \text{with } \bar{\mathbf{H}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad \text{and } \bar{\mathbf{s}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

- $\psi(0) = 1 \times 0 + 2 \times 1 + 1 \times 1 + 0 \times 0 = 3$
- $\psi(1) = 1$
- $\psi(2) = 3$

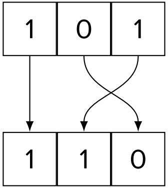
[8] U. Feige and A. Lellouche. "Quantitative Group Testing and the rank of random matrices". In: *CoRR* abs/2006.09074 (2020). arXiv: 2006.09074.

Score function : advantages

The score of the columns of $\mathbf{H}_{pub}$ provides us with a **ranking**.
This defines a **permutation** over $\mathbf{e}$ too, the **most likely** to bring t ones in the first positions.

Scores : [3, 1, 3]

Permutation : [0, 2, 1]



1 0 1

1 1 0

Bringing t ones in the first $(n-k)$ positions is sufficient.

Information-set decoding methods can then be used to recover the error vector.

Computational complexity

- Computing the dot product of two vectors is **very fast**,
- Overall cost for all columns of $\mathbf{H}_{pub}$: $\mathcal{O}((n-k)\times n) = \mathcal{O}(n^2)$
- $n = 8192$: $\approx 0.2\,\mathrm{s}$

Conclusion

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The results of the NIST PQC standardisation process are (almost) known.
With implementations comes the **threat of physical attacks**, which must be evaluated.

Interesting approach: use known cryptanalysis tools **“augmented”** with additional information.

- “Integer” syndrome decoding problem,
- Information-set decoding methods starting with a plausible permutation.

Future works:

- Improve the **recovery** of the integer syndrome,
- Apply the idea to **other problems** (and NIST PQC candidates).

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— Questions ? —